

Generalized equation of state for superfluid neutron stars

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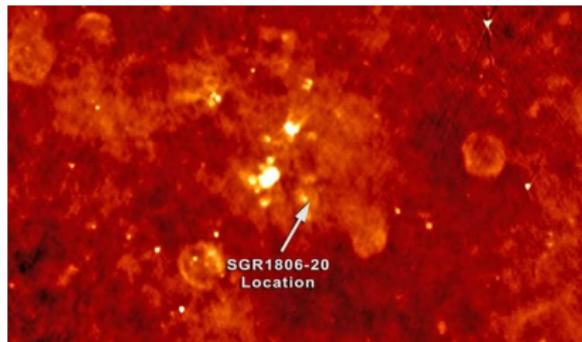
Neutron star crust and beyond, Nordita, 14-26 September 2009

Motivations

Neutron stars may contain superfluids in their interior

They evolve and can undergo instabilities triggered by

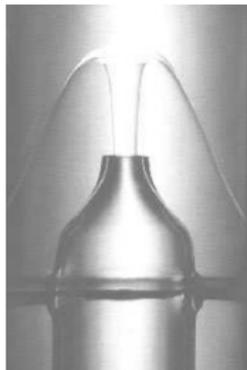
- spin-down
- thermonuclear explosions
- starquakes
- magnetic field...



Punchlines

In order to model the dynamical evolution of neutron stars, one not only needs to have a formalism describing superfluid hydrodynamics but also a suitable equation of state.

Superfluids are multi-fluid systems



One of the striking consequences of superfluidity is the possibility of having distinct dynamical components.

Example : many properties of superfluid helium can be explained by a two-fluid model



L. Tisza



L. D. Landau

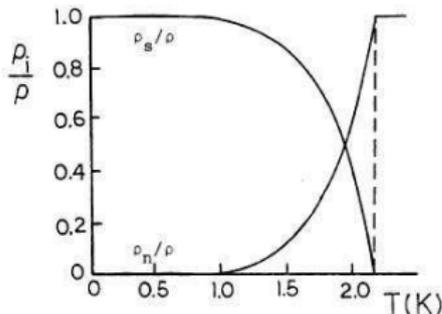
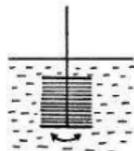
Superfluidity and entrainment

In superfluid helium at $T > 0$, the momentum \mathbf{p} and the velocity \mathbf{v} of each atom are not aligned due to the interactions between atoms and thermal excitations

entrainment

$$\mathbf{p} = m^* \mathbf{v} + (m - m^*) \mathbf{v}_N$$

\mathbf{v}_N being the velocity of quasiparticles (phonons, rotons...)

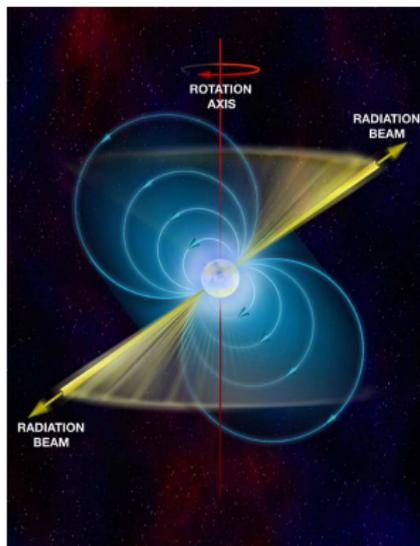


Likewise in superfluid mixtures, momentum and velocity are not proportional even at $T = 0$: this is the Andreev&Bashkin effect

Canonical model of superfluid neutron stars

Unlike neutrons, electrically charged particles are essentially locked together by the interior magnetic field on very long timescales of the order of the age of the star.

Easson, ApJ 233(1979), 711



How many fluids in neutron stars ?

at least two :

- superfluid neutrons
- superconducting “protons”

Finite- T effects and/or hyperons lead to more complicated models

Gusakov, Kantor & Haensel, Phys. Rev. C80(2009), 015803

How to obtain flow equations for fluid mixtures ?



E. Cartan



B. Carter

Variational formalism developed by Brandon Carter and coworkers, based on exterior calculus

Carter, Lect. Notes Phys. 578 (2001), Springer

Action principle

$$\mathcal{A} = \int \Lambda \{n_x^\mu\} d^4x$$

The Lagrangian density or master function Λ depends on the 4-current vectors $n_x^\mu = n_x u_x^\mu$ of the different fluids X

Newtonian superfluid models of neutron star cores

Using the traditional 3+1 space-time decomposition, the Lagrangian density of neutron-proton mixture reads

$$\Lambda = \frac{1}{2}\rho_n v_n^2 + \frac{1}{2}\rho_p v_p^2 - U(n_n, n_p) - \alpha(n_n, n_p)(\mathbf{v}_n - \mathbf{v}_p)^2$$

- $U(n_n, n_p)$ is the “static” energy density (usual equation of state)
- $\alpha(n_n, n_p)$ is a mass density associated with entrainment

In the proton rest-frame (i.e. $\mathbf{v}_p = 0$), we then find

$$\boldsymbol{\rho}_n = m_n^* \mathbf{v}_n \quad \frac{m_n^*}{m} = 1 - \frac{2\alpha(n_n, n_p)}{\rho_n}$$

General constraints on entrainment

- For non-interacting Fermi gases

$$\alpha(n_n, n_p) = 0$$

- For interacting Fermi liquids

$$\alpha(n_n, n_p) > 0$$

- Absolute stability imposes that

$$\alpha(n_n, n_p) < \frac{1}{2} m \frac{n_n n_p}{n_n + n_p}$$
$$\Rightarrow \frac{m_n^*}{m} > 1 - \frac{\rho_p}{\rho} \quad \frac{m_p^*}{m} > \frac{\rho_p}{\rho}$$

Generalized equation of state of superfluid neutron star cores

The generalized pressure can be written as

$$\Psi = P(n_n, n_p) + (\mathbf{v}_n - \mathbf{v}_p)^2 \left[\alpha(n_n, n_p) - n_n \frac{\partial \alpha}{\partial n_n} - n_p \frac{\partial \alpha}{\partial n_p} \right]$$

Microscopic models

- $npe\mu$ matter
- nucleons are treated in the mean field approximation
- leptons are treated as relativistic ideal Fermi gases

Chamel, MNRAS 388 (2008), 737

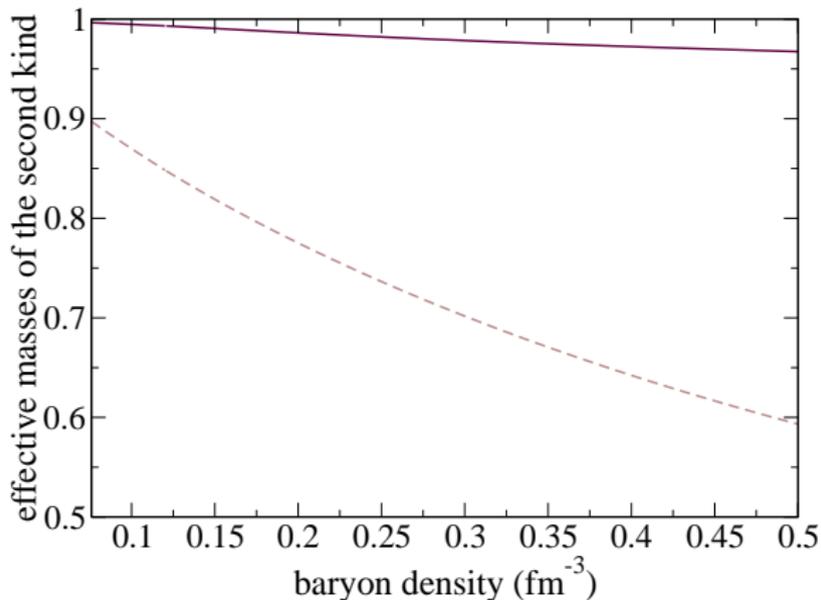
Chamel&Haensel, Phys.Rev.C 73(2006), 045802

Gusakov & Haensel, Nucl. Phys. A761 (2005),333

Comer&Joynt, Phys.Rev.D68(2003),023002

Dynamical effective masses in neutron star core

Example : Hartree-Fock equations at $T = 0$ with SLy4 force



Chamel&Haensel, Phys.Rev.C 73(2006), 045802

Relativistic superfluid hydrodynamics

The relativistic Lagrangian density is given by

$$\Lambda = \lambda_0(n_n, n_p) + \lambda_1(n_n, n_p)(x^2 - n_n n_p) + \dots$$

$$x^2 c^2 = -g_{\mu\nu} n_n^\mu n_p^\nu$$

$$n_n^2 c^2 = -g_{\mu\nu} n_n^\mu n_n^\nu$$

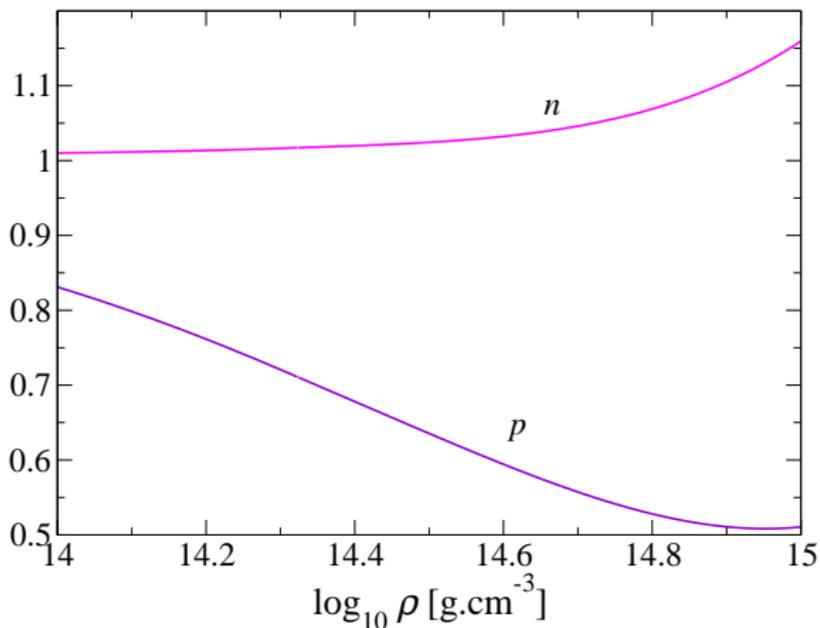
$$n_p^2 c^2 = -g_{\mu\nu} n_p^\mu n_p^\nu$$

$$\lambda_0(n_n, n_p) = -U(n_n, n_p), \quad \lambda_1(n_n, n_p) = -\frac{c^2 \alpha(n_n, n_p)}{n_n n_p}$$

Chamel, MNRAS 388 (2008), 737

Relativistic dynamical effective masses

LNS Skyrme force (fitted to Brueckner Hartree-Fock calculations with realistic nucleon-nucleon interactions)



“Chemical” equilibrium in multifluid systems

The composition is determined by the rates of transfusion processes. At equilibrium, the chemical affinity of each process vanishes $\mathcal{A}^\Xi = 0$ where

$$\mathcal{A}^\Xi \equiv - \sum_x N_x^\Xi \mathcal{E}^x$$

N_x^Ξ particle creation numbers
 \mathcal{E}^x energy per particle

For comoving particles, one can define $\mathcal{E}^x = -u^\mu \pi_\mu^x$ so that

$$\mathcal{A}^\Xi = - \sum_x N_x^\Xi \mu_x, \quad \mu_x = \frac{\partial U}{\partial n_x}$$

Carter & Chamel, Int.J.Mod.Phys.D14 (2005) 749-774.

But in multifluid systems, how to defined \mathcal{E}^x ?

Superfluid model of magnetized neutron stars



It is formally straightforward to include magnetic field in the action principle (i.e. variational formulation), even in the Newtonian framework.

*Carter, Chachoua & Chamel,
Gen.Rel.Grav.38(2006)83.*

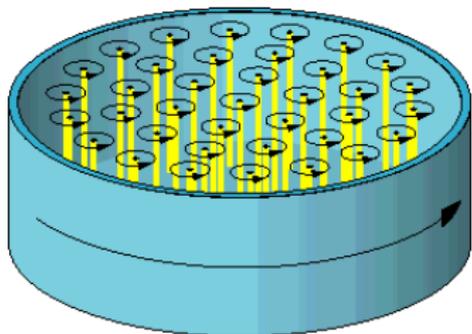
But this requires a much better understanding of superconductivity and of the dynamics of charged particles !

Entrainment and vortices

Bohr-Sommerfeld quantization rule

$$\oint \pi_{\mu}^n dx^{\mu} = Nh$$

⇒ the circulation is quantized into N vortices

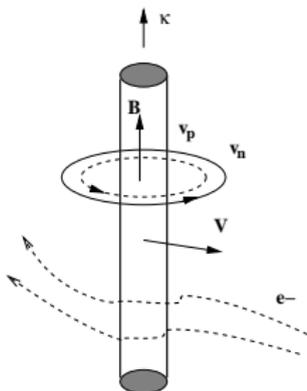


Entrainment effects have an impact on the distribution of neutron vortices

$$n_v = \frac{2m\Omega_n}{h} + \frac{1}{h}(\Omega_n - \Omega_p) \left(\varrho \frac{dm_{\star}^n}{d\varrho} + 2(m_{\star}^n - m) \right)$$

Chamel & Carter, MNRAS 368 (2006) 796.

Entrainment and vortices



picture from K. Glampedakis

Due to entrainment, neutron vortices carry a fractional magnetic quantum flux

$$\Phi_{\star} = \oint \mathbf{A} \cdot d\mathbf{l} = k\Phi_0, \quad \Phi_0 \equiv \frac{hc}{2e}$$

Alpar, Langer, Sauls, ApJ282 (1984) 533-541

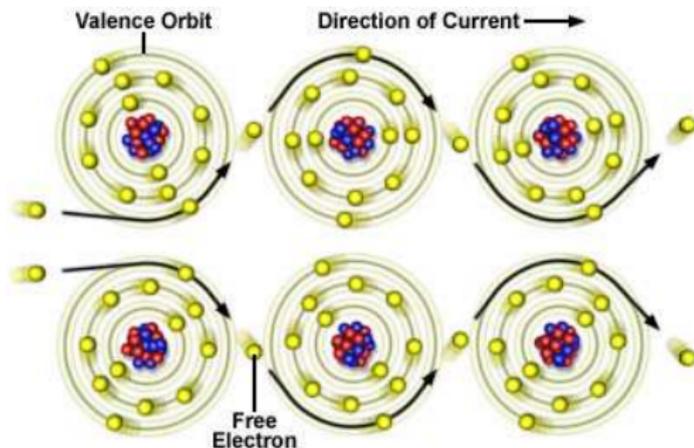
But how do vortices/fluxoids modify entrainment between superfluid neutrons and superconducting protons ?

Superfluidity and entrainment in neutron star crust

Due to the interactions with the nuclear clusters, the “free” neutrons move in the inner crust as if they had an effective mass m_n^* .

well-known effect in solid state physics

$$m_e^* \sim 1 - 2m_e$$

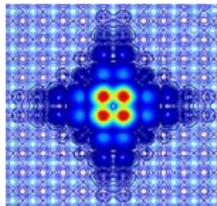
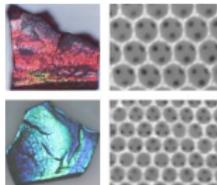


In the crust frame $\mathbf{p}_n = m_n^* \mathbf{v}_n$ therefore in another frame

$$\mathbf{p}_n = m_n^* \mathbf{v}_n + (m_n - m_n^*) \mathbf{v}_c$$

Nuclear band theory

The inner crust of neutron stars is the nuclear analog of periodic systems in condensed matter : electrons in solids, photonic and phononic crystals, cold atomic gases in optical lattice

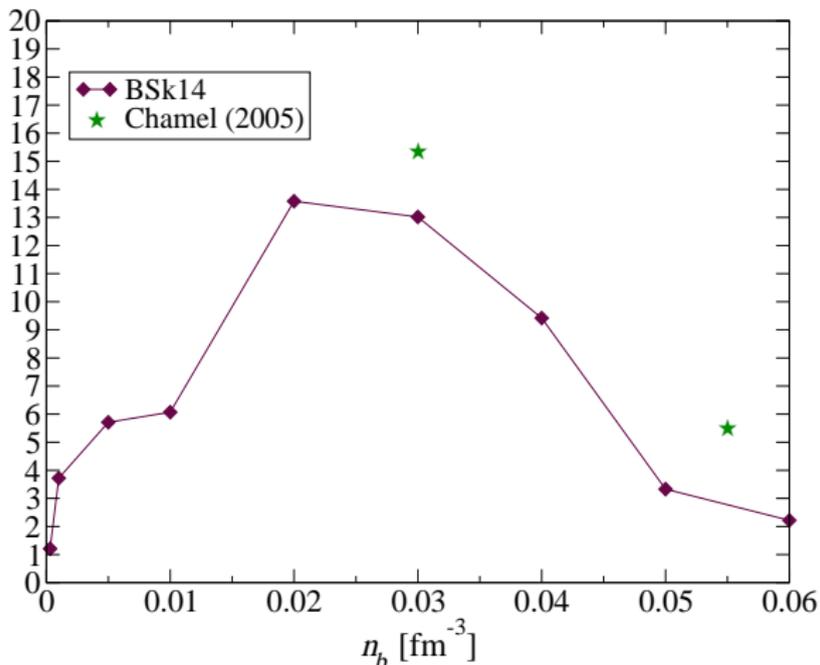


⇒ **nuclear band theory** provides a general framework for describing consistently both nuclear clusters and “free” neutrons

Chamel, Nucl.Phys.A747(2005)109.

Chamel, Nucl.Phys.A773(2006)263.

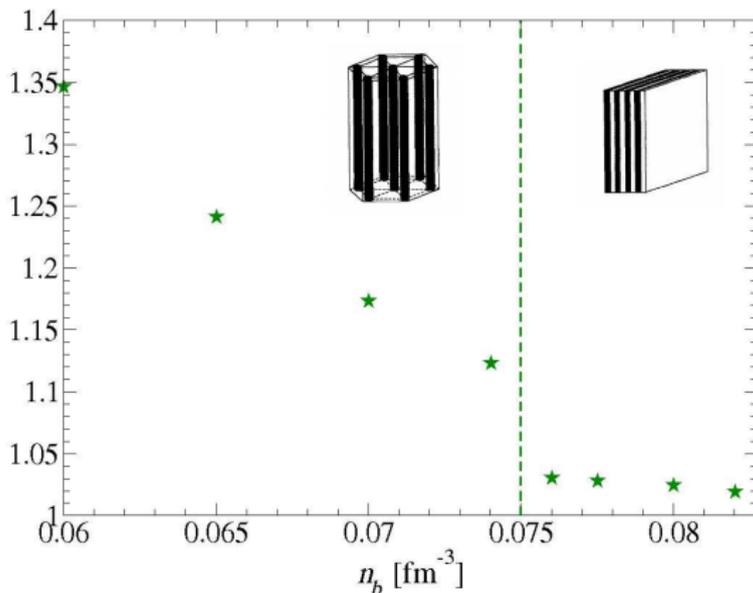
Dynamical effective neutron mass in neutron star crust



Preliminary results for BSk14 compared with previous results from Chamel, Nucl.Phys.A749, 107 (2005)

Dynamical effective mass of nuclear pastas

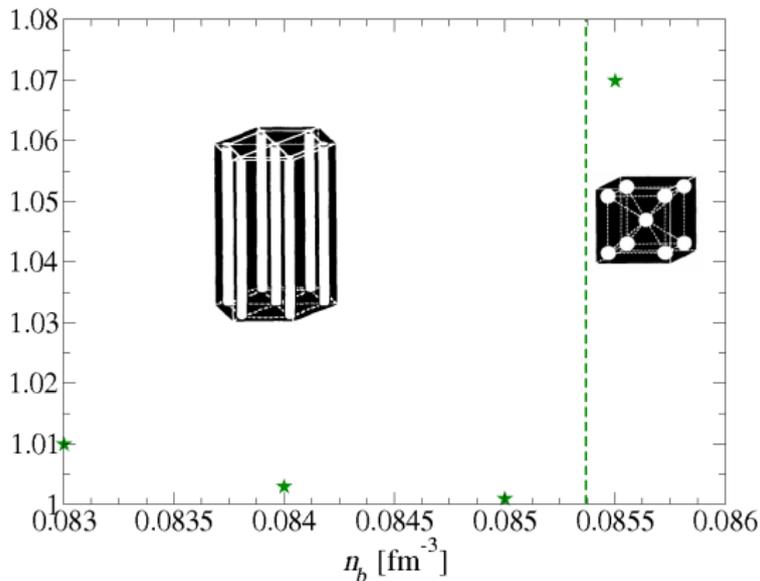
In the pasta phases, the effective mass becomes anisotropic but entrainment effects are small



Carter, Chamel, Haensel, *Nucl. Phys. A748* (2005) 675

Dynamical effective mass of nuclear pastas

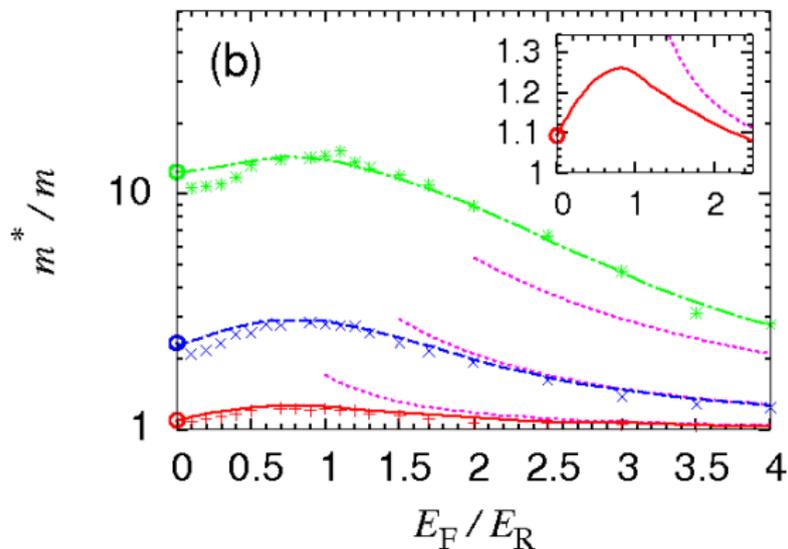
In the pasta phases, the effective mass becomes anisotropic but entrainment effects are small



Chamel, *Nucl.Phys.A749*, 107 (2005)

Entrainment effects in cold atoms

Entrainment effects are also predicted in cold atoms. Example : unitary Fermi gas in a 1D optical lattice



Watanabe et al., Phys. Rev. A78(2008),063619

Glitches and entrainment

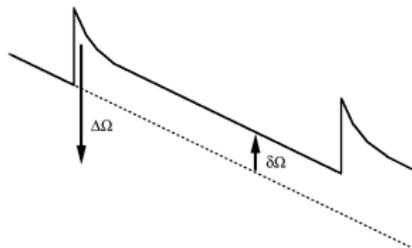
Glitches are usually interpreted as sudden transfers of angular momentum between the free neutrons in the crust and the charged particles

Baym et al., Nature 224 (1969), p673

Entrainment effects

$$J^n = I^{nn}\Omega_n + I^{nc}\Omega_c$$

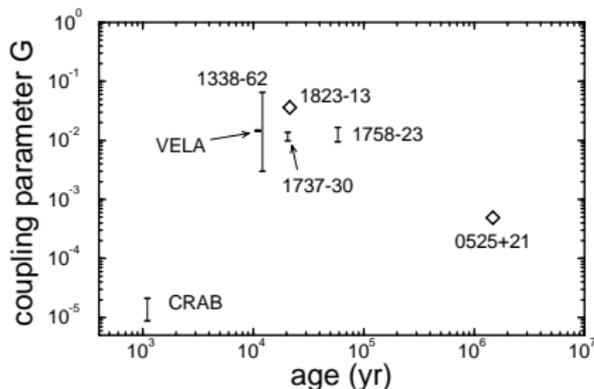
$$J^c = I^{cc}\Omega_c + I^{nc}\Omega_n$$



$$\frac{(I^n)^2}{|I^{nn}|} \geq \mathcal{G}, \quad \mathcal{G} = \frac{\Omega}{|\dot{\Omega}_{\text{av}}|} \frac{1}{\tau} \sum_i \frac{\Delta\Omega_i}{\Omega}$$

Chamel and Carter, MNRAS 368(2006), p796

Glitch constraint



Link, PRL83 (1999), 3362

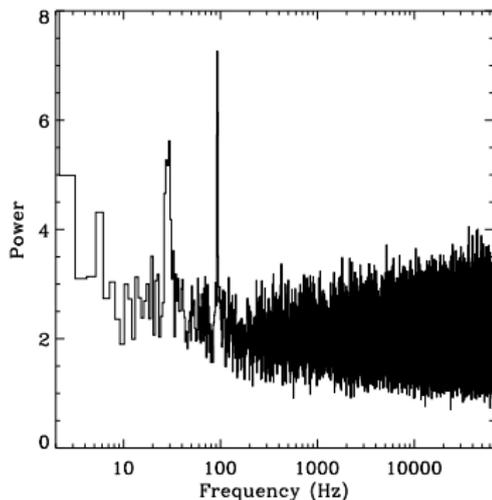
If only the crust superfluid is involved during a glitch then

$$\frac{(I^n)^2}{|I^{nn}|} < \frac{I^n}{I}$$

Entrainment makes it more difficult to explain very active glitching pulsars with crust model

Magnetar seismology

QPOs in SGRs are usually believed to be associated with seismic waves in neutron star crust.



The superfluid neutrons change the frequencies of toroidal crustal modes by $\sim 10\%$.

Samuelsson & Andersson, Class. Quant. Grav.26(2009),155016.

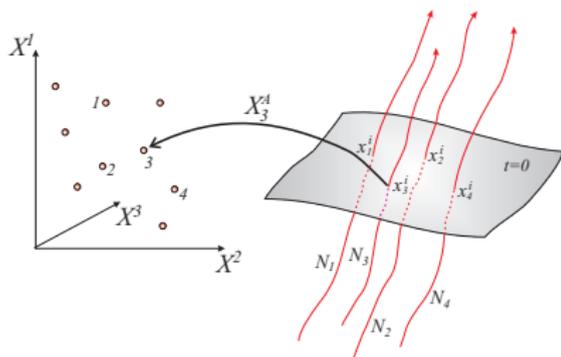
Summary

Take home message

Superfluids are not just characterized by pairing gaps but also by entrainment parameters !

- The interior of neutron stars contains at least two fluids : superfluid neutrons and “protons” (i.e. protons+leptons in the core, nuclear clusters in the crust).
- Due to mutual entrainment effects, the momentum and the velocity of each fluid are not aligned.
- Entrainment parameters have been calculated using mean field models
- Entrainment effects could leave their imprints on various phenomena like QPOs, glitches, etc.

Multi-fluid hydrodynamics



picture from Andersson&Comer

4-momentum covector

vorticity 2-form

4-force density covector

generalized Euler equations

$$n_X^\mu \varpi_{\mu\nu}^X + \pi_\nu^X \nabla_\mu n_X^\mu = f_\nu^X$$

$$\pi_\mu^X = \frac{\partial \Lambda}{\partial n_X^\mu}$$

$$\varpi_{\mu\nu}^X = \nabla_\mu \pi_\nu^X - \nabla_\nu \pi_\mu^X$$

$$f_\nu^X$$

Multi-fluid hydrodynamics

Stress-energy density tensor of the fluids can be obtained from Noether theorem

$$T^{\mu}_{\nu} = \Psi \delta^{\mu}_{\nu} + \sum_X n_X^{\mu} \pi_X^{\nu}$$

where Ψ is a generalized pressure

$$\Psi = \Lambda - \sum_X n_X^{\mu} \pi_X^{\mu}$$

In general Ψ depends on the velocities of the fluids.

Nota Bene

The above expressions are valid for any spacetime.

Neutron star crust model

- The composition of the crust is obtained using the ETFSI method. This method is a very fast approximation to Hartree-Fock calculations.
Onsi et al., Phys.Rev.C77,065805 (2008).
- The HF equation for neutrons is then solved only once using Bloch boundary conditions and the effective mass is computed from

$$m_n^* = \frac{n_n}{\mathcal{K}} \quad \mathcal{K} = \frac{1}{3} \sum_{\alpha} \int_{\text{F}} \frac{d^3k}{(2\pi)^3} \frac{1}{\hbar^2} \frac{\partial^2 \mathcal{E}_{\alpha\mathbf{k}}}{\partial k_j \partial k^i}$$

Chamel, Nucl.Phys.A747(2005)109.

Chamel, Nucl.Phys.A773(2006)263.